HL Geometry Practice Questions
03/01/2007

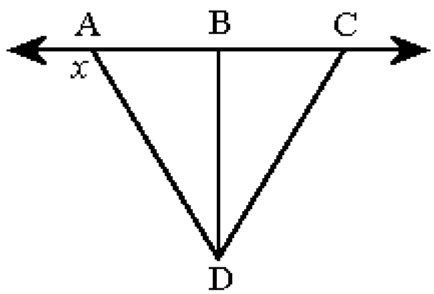
Student Name: ______________________________

Class: ______________________________

Date: ______________________________

Instructions: **Read each question carefully and select the correct answer.**

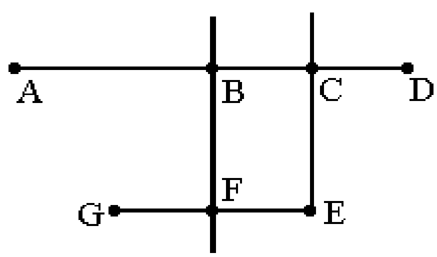
1.



$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.
If $\angle ADC = 58°$, what is the value of x?

A. 61º
B. 151º
C. 119º
D. 29º

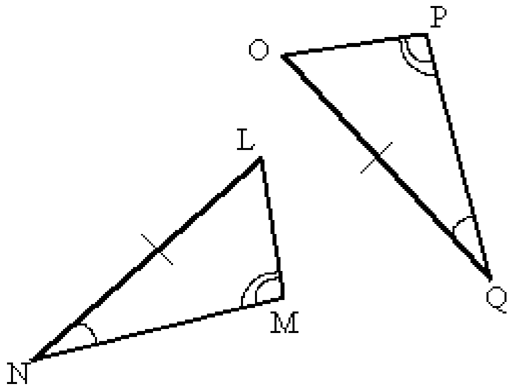
2.



$\overline{BF}$ is the perpendicular bisector of $\overline{AD}$ and $\overline{GE}$.
$\overline{CE}$ is the perpendicular bisector of $\overline{BD}$. If $GF = 5.2$ cm and $FE = CD$, what is AD?

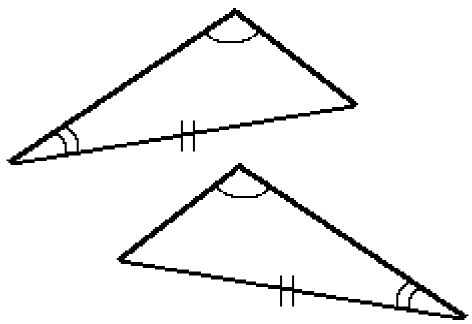
A. 10.4 cm
B. 20.8 cm
C. 5.2 cm
D. 2.6 cm

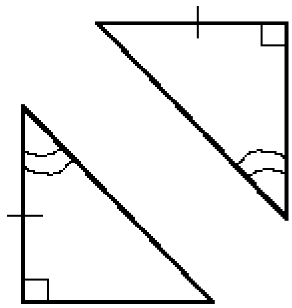
3. Using only the information presented in the diagram, determine if the following triangles are congruent, and state which congruence theorem was used.

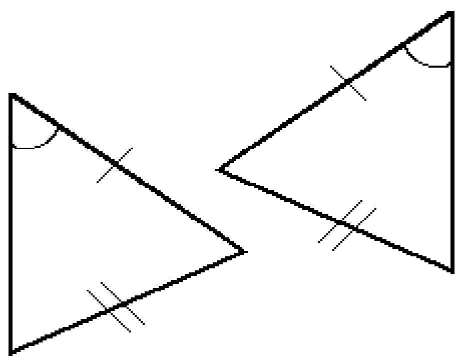


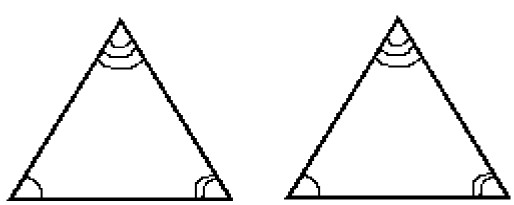
A. Triangle LMN is not congruent to triangle OPQ.
B. Triangle LMN is congruent to triangle OPQ, SAS.
C. Triangle LMN is congruent to triangle OPQ, AAS.
D. Triangle LMN is congruent to triangle OPQ, ASA.

4. Using only the information marked and your knowledge of triangle congruence postulates, which pair of triangles is congruent?

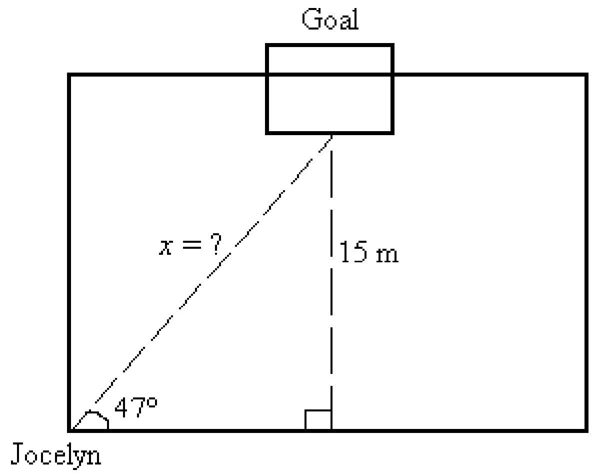
A. 

B. 

C. 

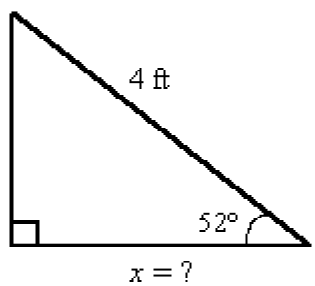
D. 

5. Jocelyn plays waterpolo. During her last game, she made a goal from the left side of the pool at half court, a length of 15 meters. She hit the ball at a 47° angle. How far did she hit the ball? Round to the nearest meter.



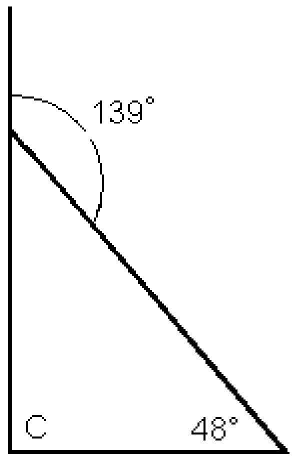
A. 22 m
B. 21 m
C. 11 m
D. 14 m

6. The handle of a lawn mower forms a 52° angle with the ground. If the handle is 4 feet long, how far away does Navik stand from the mower? Round your answer to the nearest tenth.



A. 2.5 feet
B. 3.2 feet
C. 5.1 feet
D. 6.5 feet

7. What is the measure of angle C?



A. 90º
B. 41º
C. 91º
D. 132º

8. The mall is 15 miles due south of Jodi's house. The school is 20 miles due east of the mall. What is the shortest distance from Jodi's house to the school?

A. 25 miles
B. 5 miles
C. 35 miles
D. 20 miles

9. Solve by completing the square.

$$x^2 - 18x + 100 = 0$$

A. $x = 9 \pm \sqrt{19}$

B. $x = 18 \pm 4\sqrt{14}$

C. $x = -10$ or $x = 10$
D. No Real Roots

10. Solve the equation by completing the square.

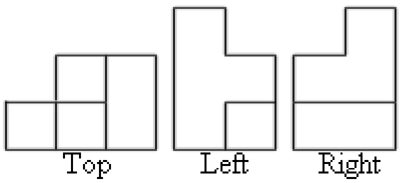
$$x^2 - 6x + 5 = 0$$

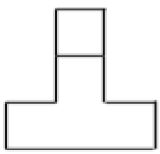
A. x = 1, 5
B. x = 5
C. x = 0, 6
D. x = 6

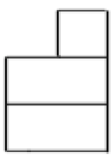
11. Which transformation was performed on the following figure?

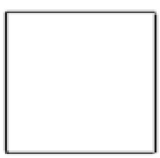
A. Rotation
B. Reflection
C. Translation
D. Dilation

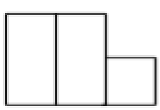
12. The following are the top, left, and right views of a building. What is the view of the front of the building?



A. 

B. 

C. 

D. 

13. John lives six miles south of Maria. Kenya lives three miles south of Maria and three miles west of Cameron. Maria lives two miles east of Samantha. Approximately how many miles does John live from Samantha?

A. six miles
B. eight miles
C. twelve miles
D. forty miles

14. The fire department had to rescue Jonathan's cat, which climbed up a tree twenty feet off of the ground. The firemen positioned their thirty foot ladder against the tree at the spot where the cat was stuck. Approximately how many feet away from the base of the tree was the ladder positioned?

A. 36 feet
B. 10 feet
C. 50 feet
D. 22 feet

15. The hike to the top of Echo Peak takes most hikers about one and a half hours. It overlooks the Lake Tahoe basin. There is a steady incline of 32º . The vertical height of the mountain is 3160 feet. What is the distance of the hike?

Angle	Sin	Cos	Tan
30°	.5000	.8660	.5774
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494

A. 3,726.42 feet
B. 5,963.39 feet
C. 4,922.21 feet
D. 1,674.48 feet

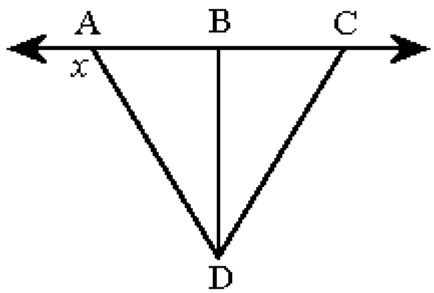
16. Flight 923 takes off from a runway at 4:35 p.m. going 95 miles an hour. The plane lifts and flies at an angle of 28º . When it gets to an altitude of 12,500 meters, the plane begins to fly parallel to the ground. Find the approximate distance the plane flies before it begins to fly parallel to the ground.

Angle	Sin	Cos	Tan
26°	.4384	.8988	.4877
27°	.4540	.8910	.5095
28°	.4695	.8829	.5317

A. 6,200 meters
B. 25,000 meters
C. 26,624 meters
D. 62,524 meters

HL Geometry Practice
Answer Key
03/01/2007

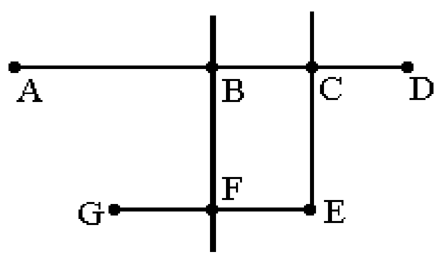
1.



$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.
If $\angle ADC = 58°$, what is the value of x?

C. 119º
Perpendicular Bisector

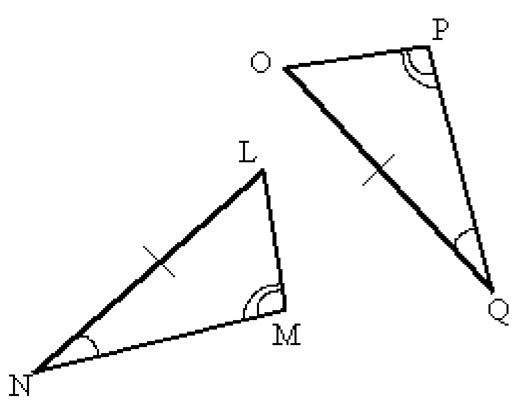
2.



$\overline{BF}$ is the perpendicular bisector of $\overline{AD}$ and $\overline{GE}$.
$\overline{CE}$ is the perpendicular bisector of $\overline{BD}$. If $GF = 5.2$ cm and $FE = CD$, what is AD?

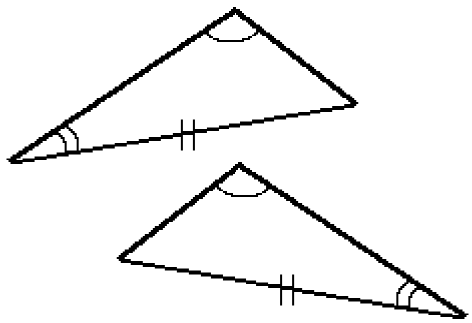
B. 20.8 cm
Perpendicular Bisector

3. Using only the information presented in the diagram, determine if the following triangles are congruent, and state which congruence theorem was used.



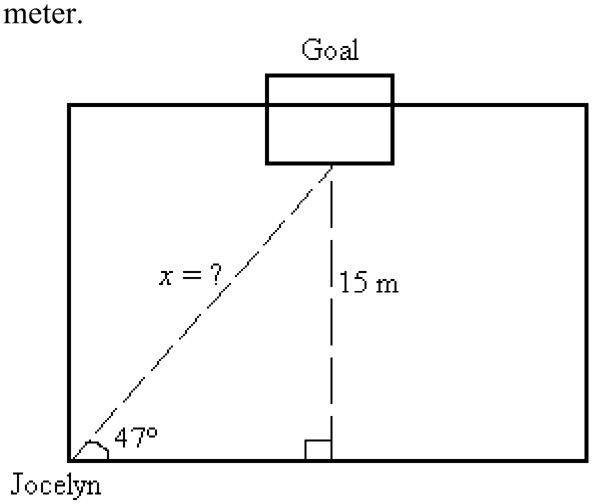
C. Triangle LMN is congruent to triangle OPQ, AAS.
Congruence (AAS/ASA/SAS) - B

4. Using only the information marked and your knowledge of triangle congruence postulates, which pair of triangles is congruent?



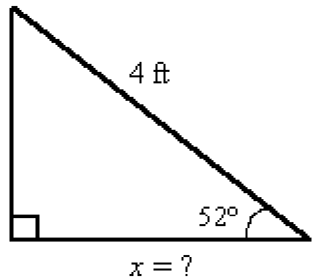
A.
Congruence (AAS/ASA/SAS) - B

5. Jocelyn plays waterpolo. During her last game, she made a goal from the left side of the pool at half court, a length of 15 meters. She hit the ball at a 47º angle. How far did she hit the ball? Round to the nearest meter.



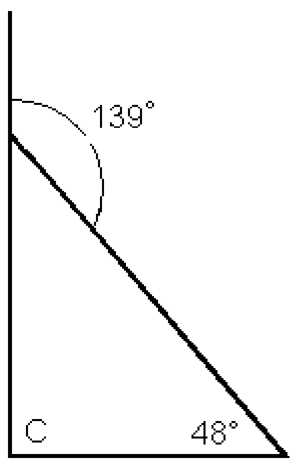
B. 21 m
Solving Right Triangles - A

6. The handle of a lawn mower forms a 52º angle with the ground. If the handle is 4 feet long, how far away does Navik stand from the mower? Round your answer to the nearest tenth.



A. 2.5 feet
Solving Right Triangles - A

7. What is the measure of angle C?



C. 91º
Triangles - B

8. The mall is 15 miles due south of Jodi's house. The school is 20 miles due east of the mall. What is the shortest distance from Jodi's house to the school?

A. 25 miles
Triangles - B

9. Solve by completing the square.

$$x^2 - 18x + 100 = 0$$

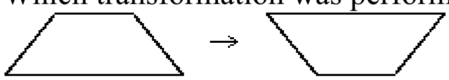
D. No Real Roots
Non-Linear Equations

10. Solve the equation by completing the square.

$x^2 - 6x + 5 = 0$

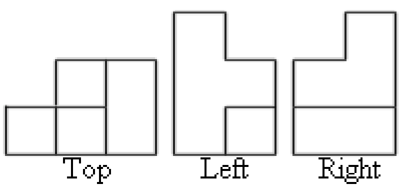
A. $x = 1, 5$
Non-Linear Equations

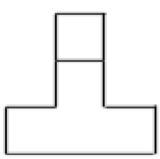
11. Which transformation was performed on the following figure?



A. Rotation
Spatial Relationships - C

12. The following are the top, left, and right views of a building. What is the view of the front of the building?



A. 
Spatial Relationships - C

13. John lives six miles south of Maria. Kenya lives three miles south of Maria and three miles west of Cameron. Maria lives two miles east of Samantha. Approximately how many miles does John live from Samantha?

A. six miles
Pythagorean Theorem

14. The fire department had to rescue Jonathan's cat, which climbed up a tree twenty feet off of the ground. The firemen positioned their thirty foot ladder against the tree at the spot where the cat was stuck. Approximately how many feet away from the base of the tree was the ladder positioned?

D. 22 feet
Pythagorean Theorem

15. The hike to the top of Echo Peak takes most hikers about one and a half hours. It overlooks the Lake Tahoe basin. There is a steady incline of 32º . The vertical height of the mountain is 3160 feet. What is the distance of the hike?

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B. 5,963.39 feet
Solving Right Triangles - B

16. Flight 923 takes off from a runway at 4:35 p.m. going 95 miles an hour. The plane lifts and flies at an angle of 28º . When it gets to an altitude of 12,500 meters, the plane begins to fly parallel to the ground. Find the approximate distance the plane flies before it begins to fly parallel to the ground.

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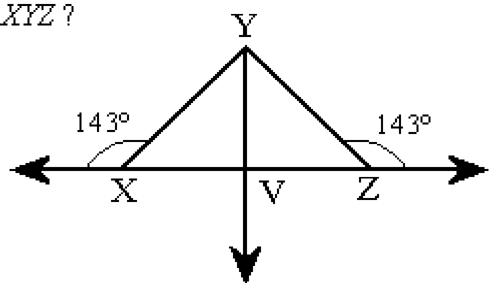
C. 26,624 meters
Solving Right Triangles - B

HL Geometry Practice
03/01/2007

Perpendicular Bisector-
A <u>perpendicular bisector</u> is any line, segment, or ray that forms a 90º angle to a segment at its midpoint. In other words, it cuts the segment in half and forms right angles at the point of intersection.

Example 1:

$\overrightarrow{YV}$ is the perpendicular bisector of $\overline{XZ}$. What is $m\angle XYZ$?



(1) $m\angle YXV = 180° - 143° = 37°$

(2) $m\angle XYV = 180° - (90° + 37°)$

$= 180° - 127° = 53°$

(3) $m\angle ZYV = 53°$

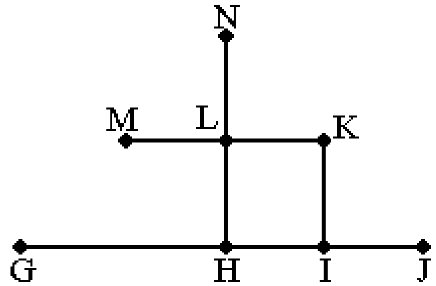
(4) $m\angle XYZ = m\angle XYV + m\angle ZYV$

$= 53° + 53° = 106°$

<u>Step 1:</u> Determine the measure of $\angle YXV$. Recall that a straight line has a total of 180º .
<u>Step 2:</u> Recall that $\angle YVX$ must be a right angle, by definition of segment YV being a perpendicular bisector to segment *XZ*. The measures of two of the angles of triangle *XYV* are now known to be 90º and 37º . The third angle, $\angle XYV$ can be determined by subtracting the sum of these two angles from 180º , the total number of degrees of the interior angles of any triangle.
<u>Step 3:</u> If Steps 1 and 2 are repeated on triangle *VYZ*, it's easy to see the same angle measure will be the result. Therefore, $\angle ZYV$ is also equal to 53º .
<u>Step 4:</u> $\angle XYZ$ is equal to $\angle XYV + \angle ZYV$.

Answer: $m\angle XYZ = 106º$

Example 2:



$\overline{NH}$ is the perpendicular bisector of $\overline{GJ}$ and $\overline{MK}$. $\overline{KI}$ is the perpendicular bisector of $\overline{HJ}$. If $MK = 1.9$ m and $ML = IJ$, what is GI?

(1) $ML = LK = 1.9 \div 2 = 0.95$

(2) $ML = IJ = LK = 0.95$

(3) $HI = IJ = 0.95$

(4) $HJ = HI + IJ = 1.9$

(5) $GH = HJ = 1.9$

(6) $GI = GH + HI = 1.9 + 0.95$
$= 2.85$

Step 1: Since segment *NH* is the perpendicular bisector of segment *MK*, it cuts the segment into two parts of equal length. Therefore, *ML* = *LK*. Since *MK* = 1.9, divide by two to determine *ML* and *LK*.
Step 2: Segment *ML* is given to be the same length as segment *IJ* and in Step 1 it is known that segment *ML* is also the same length as segment *LK*. Therefore, *ML* = *IJ* = *LK*.
Step 3: Since segment *KI* is the perpendicular bisector of segment *HJ*, it cuts the segment into two parts of equal length. Therefore, *HI* = *IJ*.
Step 4: To determine the length of segment *HJ*, we must add the lengths of segments *HI* and *IJ*.
Step 5: Because segment *NH* is the perpendicular bisector of segment *GJ*, it also cuts the segment into two parts of equal length. Therefore, *GH* = *HJ*.
Step 6: To determine the length of segment *GI*, we must add the lengths of segments *GH* and *HI*.

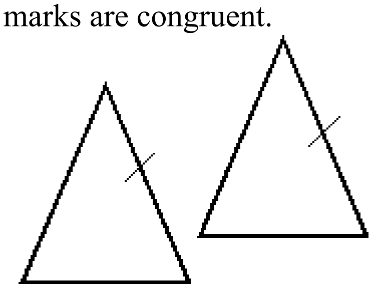
Answer: *GI* = 2.85 m

Congruence (AAS/ASA/SAS) - B

Engineers and people in the construction field use triangle congruency on a daily basis to make sure that things like rafters are congruent so the roof of a house does not sink in. This study guide will introduce students to postulates, which are used to determine triangle congruency. A postulate is a statement which is taken to be true without proof.

Review of Triangle Congruency Symbols

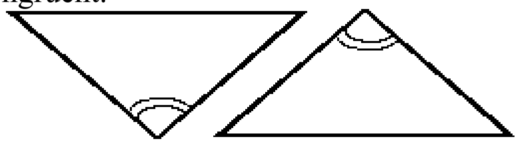
Tick marks are used to show that sides are congruent. All sides in a diagram that are marked with the same number of tick marks are congruent.



These sides are congruent.

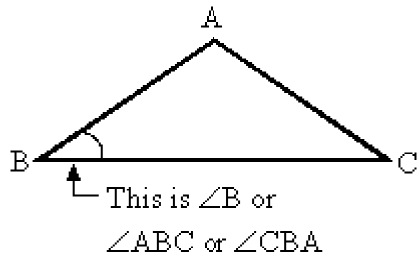
Arcs are used to show that angles are congruent. All angles in a diagram that are marked with the same

number of arcs are congruent.



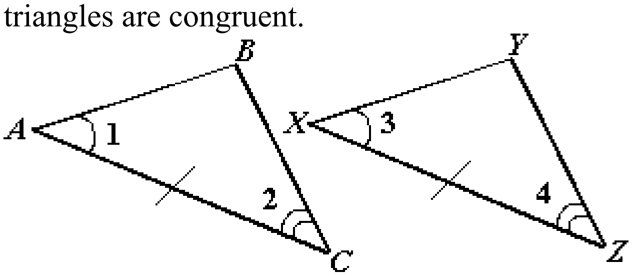
These angles are congruent.

Angles in a triangle are named by their <u>vertex</u>, the point at which two line segments meet. They are also named with the vertex in the middle.



Postulates for Congruency:

<u>ASA (Angle, Side, Angle) Postulate</u>: If two angles and the included side (the side between the two angles) of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



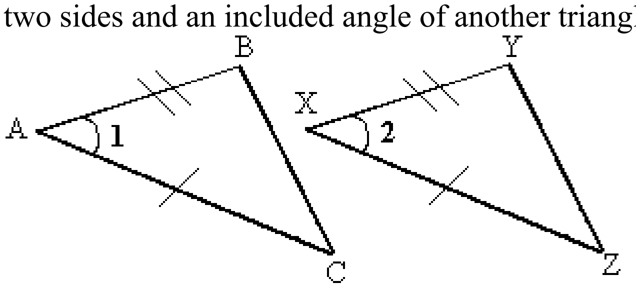
$\angle 1 \cong \angle 3$, $\overline{AC} \cong \overline{XZ}$, $\angle 2 \cong \angle 4$

Angle Side Angle

A S A

$\cong$ is the symbol for *congruent*.

<u>SAS (Side, Angle, Side) Postulate</u>: If two sides and an included angle of one triangle are congruent to two sides and an included angle of another triangle, then the triangles are congruent.

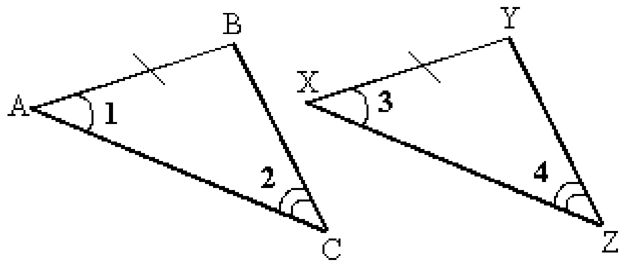


$\overline{AB} \cong \overline{XY}$, $\angle 1 \cong \angle 2$, $\overline{AC} \cong \overline{XZ}$

Side Angle Side

S A S

<u>AAS (Angle, Angle, Side) Postulate</u>: If two angles and a non-included side of one triangle are congruent to two angles and a non-included sides of another triangle, then the triangles are congruent.

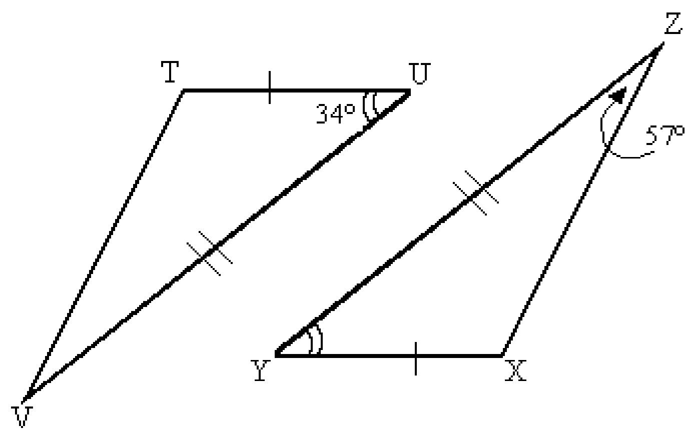


$\angle 1 \cong \angle 3,\ \angle 2 \cong \angle 4,\ \overline{AB} \cong \overline{XY}$

Angle	Angle	Side
A	A	S

Determining that Triangles are Congruent:

Example 1: What is the measure of ∠YXZ?



(1) $\overline{TU} \cong \overline{XY},\ \angle TUV \cong \angle XYZ,\ \overline{UV} \cong \overline{YZ}$

Side	Angle	Side

(2) $\triangle TUV \cong \triangle XYZ$ by SAS

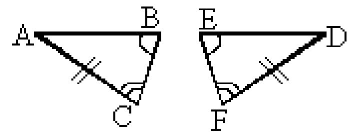
(3) $m\angle TUV = m\angle XYZ$, so $m\angle XYZ = 34°$

(4) $m\angle YXZ = 180° - 57° - 34° = 89°$

<u>Step 1:</u> Write out all the information given in the diagram.
<u>Step 2:</u> Determine if the triangles are congruent and, if congruent, by what postulate? These triangles are congruent by the SAS postulate.
<u>Step 3:</u> Since the triangles are congruent, m∠*TUV* equals m∠*XYZ*. m∠*XYZ* = 34º .
<u>Step 4:</u> Remember that the sum of the 3 angles of a triangle is 180º . To determine m∠*YXZ*, subtract 180 - 57 - 34 = 89.

Answer: 89º

Example 2: Using only the information presented in the diagram, determine if the following triangles are congruent and state which congruence theorem was used.



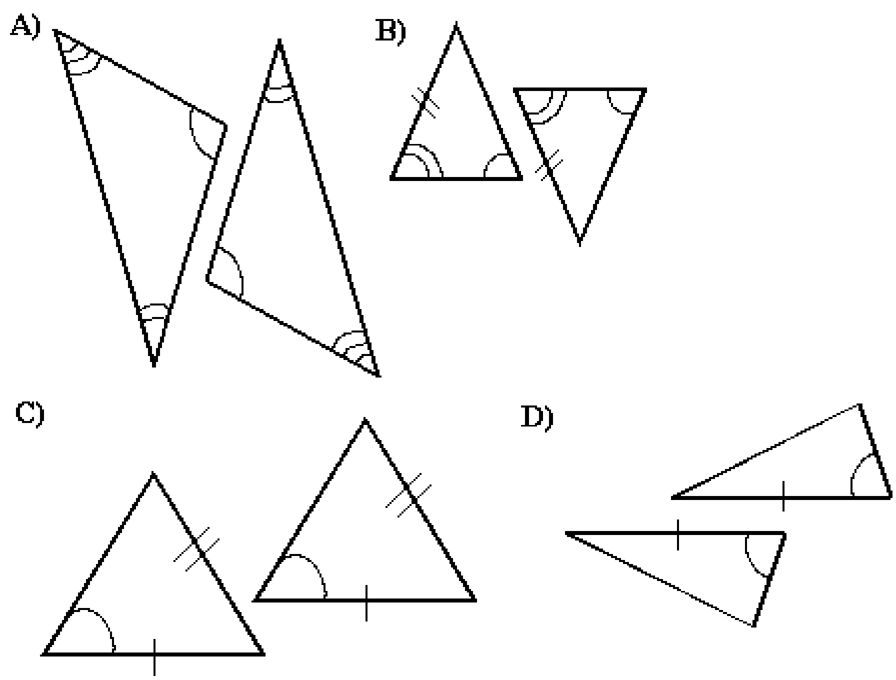
(1) $\angle ABC \cong \angle DEF$, $\angle BCA \cong \angle EFD$, $\overline{AC} \cong \overline{DF}$
angle angle side

(2) $\triangle ABC \cong \triangle DEF$ (AAS)
two $\angle$'s and a non-included side of one $\triangle \cong$
two $\angle$'s and a non-included side of another $\triangle$

<u>Step 1:</u> State the given information as shown in the diagram.
<u>Step 2:</u> The triangles are congruent by the AAS postulate.

Answer: The triangles are congruent by the AAS postulate.

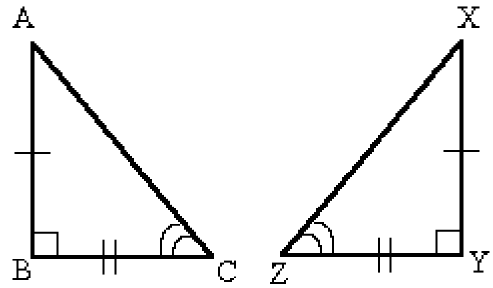
Example 3: Using only the information below, and your knowledge of triangle congruence postulates, which pair of triangles are congruent?



A is not correct. The angles of one triangle are congruent to the angles of another. This proves similarity, not congruence.
B is correct. Two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle. The triangles are congruent by the AAS postulate.
C is not correct. SSA is not true for every triangle, that is why it is not a postulate.
D is not correct. There is not enough information to prove congruency.

Example 4:

Using only the information presented, which of the following can be used to prove that $\triangle ABC \cong \triangle XYZ$?



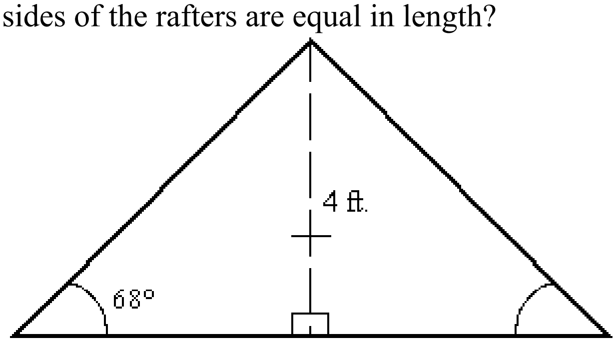
1) AAS
2) SSS
3) ASA
4) SAS

A. 2 and 4
B. 1 only
C. 1, 3, and 4
D. 4 only

A is not correct. Although SAS does prove that the triangles are congruent, SSS cannot be proven using only what is presented in the drawing.
B is not correct. While AAS does prove that the triangles are congruent, SAS does too, and it is not part of option B.
C is correct. AAS, ASA, and SAS will all prove that the two triangles are congruent using only what is presented in the drawing.
D is not correct. SAS is not the only postulate that will prove these triangles congruent using only what is presented in the drawing.

Answer: C

Example 5: Jacob is constructing rafters for a house. The middle piece of wood stands 4 feet tall and is perpendicular to the base. He attaches two pieces of wood at the top of the middle piece which create a 68° angle with the base on either side. Using only the information presented, which postulate would be used to prove that the sides of the rafters are equal in length?



A) ASA **B)** SSS **C)** AAS **D)** SAS
A) is not correct. The side that is congruent is not included between the congruent angles.
B) is not correct. Only one side is marked congruent.
C) is correct. The base angles are congruent, the right angles are congruent, and the middle piece of wood is congruent to itself. The triangles are congruent by the AAS postulate.
D) is not correct. Only one side is marked congruent.

The following is a fun activity to help reinforce the concept of triangle congruency. Take a walk with the student and observe where triangles occur in the neighborhood. Look at houses, buildings, parks, and landscapes and pick out triangles that look congruent. If possible, take a ruler or tape measure and a

protractor along to a public place or a place where it will not matter if measurements are taken. Take measurements of the sides and angles of triangular objects in pairs. Verify that they are congruent by comparing their measures. After the walk, have the student build a roof rafter out of toothpicks. Use a ruler and a protractor to measure the sides and angles. Verify that all the triangles are the same using the congruency postulates. Ask the student questions like, "What would happen if the triangles were not congruent? What would happen if engineers built bridges with triangles that were not congruent?" Have the student research on the Internet or at the library why congruent triangles are important in real life situations.

Solving Right Triangles - A

This study guide will focus on solving real world problems using right triangles that involve trigonometric concepts. Real world problems are often difficult for students. It may be beneficial to confirm that the student is comfortable with trigonometric functions and the right triangle outside of a real world context. Begin by reviewing the definitions and calculation procedures for determining sine, cosine, and tangent with the student. Calculator use is recommended.

Important Terms:

- A right triangle is a polygon with three sides, where one of its angles is exactly 90º .
- A hypotenuse is the side opposite the right angle (the slant or longest side) in a right triangle.
- The two sides of a right triangle that are not the hypotenuse are called legs.
- The sum of the interior angles of a triangle is 180º . Since one angle of a right triangle is 90º , the other two angles must each be less than 90º , which makes them acute.

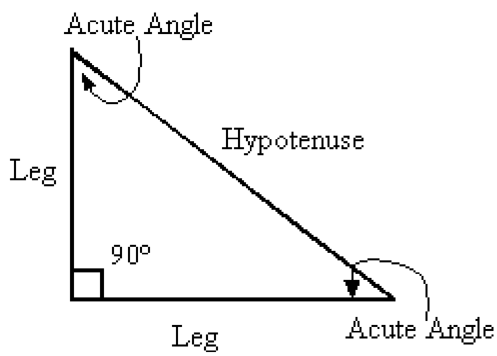


Figure 1

Consider the triangle below. Using angle C as a reference, line segment AB is the side opposite the angle, segment BC is the side adjacent to the angle, and segment AC is the hypotenuse.

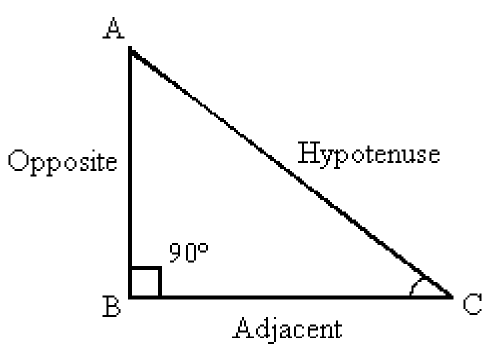


Figure 2

Determining the Sine of an Angle:

$$\text{Sine of Angle} = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$$

The sine of angle C in Figure 2 is as follows:

$$\sin C = \frac{\overline{AB}}{\overline{AC}}$$

Determining the Cosine of an Angle:

$$\text{Cosine of Angle} = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$$

The cosine of angle C in Figure 2 is as follows:

$$\cos C = \frac{\overline{BC}}{\overline{AC}}$$

Determining the Tangent of an Angle:

$$\text{Tangent of Angle} = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$$

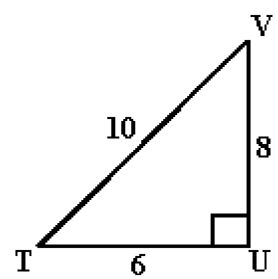
The tangent of angle C in Figure 2 is as follows:

$$\tan C = \frac{\overline{AB}}{\overline{BC}}$$

The following is a helpful way to remember the trigonometric functions. It is pronounced "So"-"Ca"-"Tow-a".

SOH	**CAH**	**TOA**
$\sin = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\tan = \dfrac{\text{opposite}}{\text{adjacent}}$

Example 1: Find the sine of ∠T in the triangle below.



1) $\text{Sine of angle T} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

2) $\sin T = \dfrac{8}{10}$

3) $\sin T = \dfrac{4}{5}$

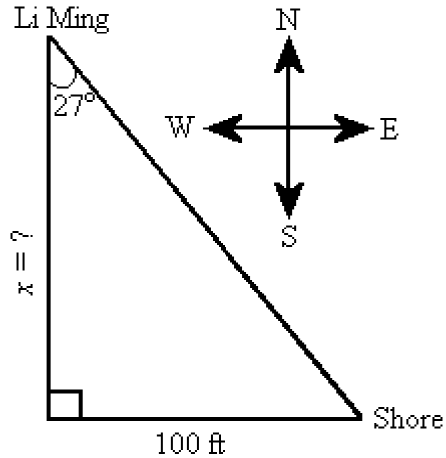
<u>Step 1:</u> Determine which components to use for the sine of an angle (opposite and hypotenuse).
<u>Step 2:</u> Substitute 8 for *length of opposite* and 10 for *length of hypotenuse* into the formula.
<u>Step 3:</u> Reduce.

Answer: $\frac{4}{5}$

Right Triangles in Story Problems:

In story problems dealing with real world situations, the measure of the reference angle and the measure of one side of the triangle are usually given. It is the job of the student to recognize which trigonometric function is needed and then substitute the given information into the formula and solve for the missing side. See the example below.

Example 2: In the beginning of her morning surf session, Li Ming paddled due west 100 feet from shore, then the current carried her due north. At his point, Li Ming was at a 27º angle with the spot she started on the shore. How far north did the current take her? Round your answer to the nearest tenth of a foot.



1) $\text{tangent of angle} = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

2) $\tan 27° = \dfrac{100}{x}$

3) $(x)\tan 27° = \dfrac{100}{x}(x)$

4) $(x)\tan 27° = 100$

5) $\dfrac{(x)\cancel{\tan 27°}}{\cancel{\tan 27°}} = \dfrac{100}{\tan 27°}$

6) $x = \dfrac{100}{0.5095}$

7) $x \approx 196.3$

Step 1: Determine which trigonometric function is needed to solve the problem. The only acute angle given in the problem is 27º , so the trigonometric function must use that angle. The length of the adjacent side is needed and the length of the opposite side is given, so the tangent function is the correct function to use.
Step 2: Substitute 27º for the angle and 100 for the length of the opposite side.
Step 3: Multiply both sides of the equation by *x* to eliminate the denominator.
Step 4: Simplify.
Step 5: Divide both sides of the equation by tan 27º to isolate the *x*.
Step 6: Have the student use a calculator or a trigonometry table (generally found in the back of the text book) to determine that the tangent of 27º is approximately 0.5095. Substitute this value into the equation in place of tan 27º .
Step 7: Divide 100 by 0.5095 and round to the nearest tenth to get 196.3.

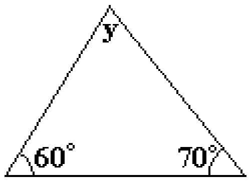
Answer: Li Ming was carried north 196.3 feet by the current.

Have the student research where right triangles are used in everyday situations. Resources that may prove useful include magazines, newspapers, educational television shows, and the Internet.

Triangles - B

A triangle is a three-sided figure that has three angles. The sum of the three angle measures of a triangle is 180º . When we are given the other two angle measures, we can find the unknown angle measure by adding the two known angle measures and subtracting that number from 180º .

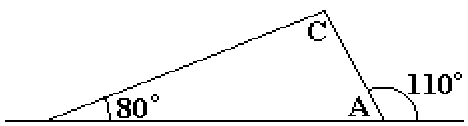
Example 1: What is the value of y?



Add the two known angles: 60º + 70º = 130º
Since the sum of the angle measures must equal 180º , subtract: 180º - 130º = 50º .

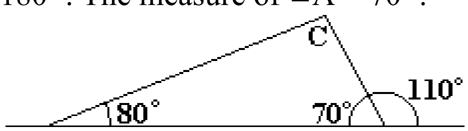
The unknown angle measure is 50º .

Example 2: What is the measure of ∠C?



(1) 110º + m∠A = 180º ; 180º - 110º = 70º ; m∠A = 70º
(2) 180º - (70º + 80º) = ∠C
(3) 180º - 150º = 30º

Step 1: We must find the measure of ∠A. We know that the sum of the angles on a straight line equal 180º . The measure of ∠A = 70º .



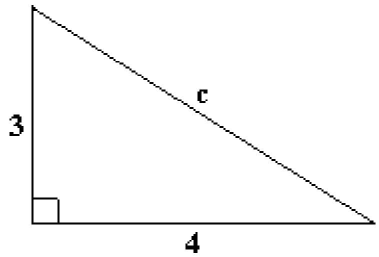
Step 2: Subtract the sum of the angles (70º + 80º = 150º) from 180º .
Step 3: The measure of ∠A is 30º

The Pythagorean Theorem allows us to determine the lengths of sides of a right triangle. (A right triangle is a triangle with a 90º angle). The Pythagorean Theorem states that the square of the hypotenuse (the longest side, and side opposite the 90º angle) of a right triangle is equal to the sum of the squares of the legs (the two shorter sides) of that triangle. The formula in written form is:

$$a^2 + b^2 = c^2$$

where a and b are the short sides and c is the long side (the hypotenuse).

Example 3: The lengths of two legs of a right triangle are 4 and 3. What is the length of the hypotenuse c?



$(1)\ 4^2 + 3^2 = c^2$

$(2)\ 16 + 9 = c^2$

$(3)\ 25 = c^2$

$(4)\ \sqrt{25} = \sqrt{c^2}$

$(5)\ 5 = c$

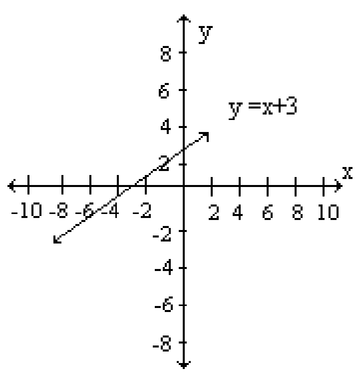
Step 1: a = 4 and b = 3. Substitute these values into the Pythagorean theorem.
Step 2: Evaluate the values of the square terms. 4 x 4 = 16 and 3 x 3 = 9.
Step 3: Add 16 and 9 to get 25.
Step 4: Take the square root of both sides of the equation to isolate the variable c.
Step 5: We only need the positive value of the square root because we are talking about a distance.

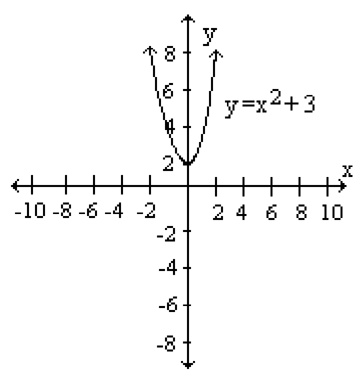
The hypotenuse of the triangle is 5.

Non-Linear Equations

A non-linear equation is an equation whose graph is **not** a straight line.

An example of a linear equation (graph is a straight line) is y = x + 3 and an example of a non-linear equation is $y = x^2 + 3$. These two equations are graphed below.





A basic rule to follow to determine whether an equation is linear or non-linear is that non-linear equations have variables that have powers other than one and linear equations have powers equal only to one.

Simplifying Square Roots:

Before we can begin solving non-linear equations, we must discuss how to simplify square roots. When a number is multiplied by itself the product is the square of the number. A square root of a number is a factor that when multiplied by itself equals the number. For example: 2 x 2 = 4, so 4 is the square of 2 also, since 4 = 2 x 2, 2 is a square root of 4. Another square root of 4 is -2 because -2 x -2 = 4. Numbers that have a rational number as their square root are called perfect squares. Examples of perfect squares are: 9 (square root is 3), 16 (square root is 4), 25 (square root is 5) and 9/25 (square root is 3/5).

The notation for a square root is this symbol: $\sqrt{\ }$.
$\sqrt{16}$ is read "the square root of 16" or "radical 16".
To simplify a square root, we first determine two factors that multiply to make the whole number. One of these two factors should be a perfect square, preferably the largest perfect square that is a factor of the number. Then we take the square root of the perfect square factor and place that number in front of the radical symbol.

Example 1: Simplify.

$\sqrt{72}$

(1) $\sqrt{72} = \sqrt{36 \cdot 2}$

(2) $\sqrt{72} = \sqrt{36} \cdot \sqrt{2}$

(3) $\sqrt{72} = 6\sqrt{2}$

Step 1: Rewrite the problem as the square root of two factors of 72. Remember, one of the factors should be the largest perfect square that is a factor of 72. In this case 36 and 2 were used because 36 times 2 equals 72 and 36 is a perfect square.
Step 2: Now the problem can be rewritten as the square root of 36 times the square root of 2.
Step 3: Determine the square root of 36 (which is 6) and multiply it by radical 2. Since 2 does not have a factor that is a perfect square, radical 2 does not change.

Example 2: Another way to simplify $\sqrt{72}$

(1) $\sqrt{72} = \sqrt{9 \cdot 8}$

(2) $\sqrt{72} = \sqrt{9 \cdot 4 \cdot 2}$

(3) $\sqrt{72} = \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{2}$

(4) $\sqrt{72} = 3 \cdot 2 \cdot \sqrt{2}$

(5) $\sqrt{72} = 6\sqrt{2}$

Step 1: It is possible to simplify a square root if the largest perfect square factor is not known. Once again, rewrite the problem as the square root of two factors of 72. Make sure one of these two factors is a perfect square. In this case 9 and 8 were used because 9 is a perfect square and 9 times 8 equals 72.
Step 2: Since 8 has a factor that is a perfect square (4), the problem must be rewritten as the product of these three factors of 72. 9 x 4 x 2 = 72.
Step 3: Since 9 and 4 are perfect squares and 2 does not have a perfect square factor, the problem can be rewritten as the square root of 9 times the square root of 4 times the square root of 2.
Step 4: Determine the square root of 9 (which is 3), multiply it by the square root of 4 (which is 2), and multiply them both by radical 2.
Step 5: Finally, multiply 3 and 2 to get 6. The 6 is multiplied by radical 2 to obtain the final answer: $6\sqrt{2}$.

Solving Non-Linear Equations:

Non-Linear equations can be solved in much the same way as linear equations. The goal of solving a non-linear equation is to isolate the variable on one side of the equal sign.

Example 3: Solve the following equation.

$$-2x^2 + 18 = 3x^2 - 12$$

$$\begin{array}{ll} & -2x^2+18=3x^2-12 \\ (1) & \underline{+2x^2 \qquad +2x^2} \\ & 18 = 5x^2 - 12 \\ (2) & \underline{+12 \qquad\quad +12} \\ (3) & \frac{30}{5} = \frac{5x^2}{5} \\ (4) & x^2 = 6 \\ (5) & \sqrt{x^2} = \sqrt{6} \\ (6) & x = \pm\sqrt{6} \end{array}$$

Step 1: Add $2x^2$ to each side of the equation, placing it under its like terms. This will get all of the x^2 terms on the same side of the equal sign.
Step 2: Add 12 to each side of the equation, placing it under its like terms. This will get all of the terms that do not have an x^2 on the opposite side of the equal sign as the terms with x^2.
Step 3: Divide each side of the equation by 5. This will get the x^2 by itself on one side of the equal sign.
Step 4: $6 = x^2$ can be rewritten as $x^2 = 6$, because the two terms will be equal no matter which is written first.
Step 5: In order to solve the equation, we must have x with no exponent. To eliminate the power of 2 from x^2, we must take the square root of each side of the equation.
Step 6: All terms have a positive and a negative square root, so we must put the $\pm$ symbol in front of the answer. Six does not have a perfect square factor, so it cannot be simplified any further.

Answer: $x = \pm\sqrt{6}$

Example 4: Solve the following equation.

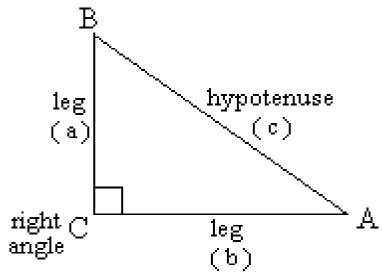
$$10x^2 + 9 = 8x^2 + 33$$

$$\begin{array}{ll} & 10x^2 + 9 = 8x^2 + 33 \\ (1) & \underline{-8x^2 \qquad -8x^2} \\ & 2x^2 + 9 = 33 \\ (2) & \underline{\quad -9 \quad -9} \\ (3) & \frac{2x^2}{2} = \frac{24}{2} \\ (4) & x^2 = 12 \\ (5) & \sqrt{x^2} = \pm\sqrt{12} \\ (6) & x = \pm 2\sqrt{3} \end{array}$$

Step 1: Subtract $8x^2$ from each side of the equation, placing it under its like terms.
Step 2: Subtract 9 from each side of the equation, placing it under its like terms.
Step 3: Divide each side of the equation by 2 to isolate the x^2.
Step 4: 24 divided by 2 is 12, so $x^2 = 12$.
Step 5: Take the square root of each side of the equation to eliminate the power of 2 on the x.
Step 6: Simplify $\sqrt{12}$. Remember that every term has two square roots, so the $\pm$ symbol must be placed in front of the answer.

Using the Pythagorean Theorem to Solve Non-Linear Equations:

The Pythagorean Theorem can be used to determine the length of a missing side of a right triangle. A right triangle is a triangle that has one right (90º) angle. A 90º angle is marked in a triangle with a box in the angle. The Pythagorean Theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. The hypotenuse of a right triangle is the side of the triangle opposite the right angle. The other two sides of the triangle are called the legs.

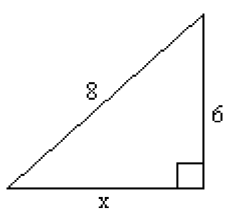


The Pythagorean Theorem states:

$$a^2 + b^2 = c^2$$

In the Pythagorean Theorem, 'c' represents the length of the hypotenuse and 'a' and 'b' represent the lengths of the legs of the right triangle. The Pythagorean Theorem only works for right triangles.

Example 5: Use the Pythagorean Theorem to solve for x.



(1) $c^2 = a^2 + b^2$
$a = x, b = 6, c = 8$

(2) $8^2 = x^2 + 6^2$

(3) $64 = x^2 + 36$

(4) $64 = x^2 + 36$
$-36 \quad -36$
$28 = x^2$

(5) $\sqrt{x^2} = \sqrt{28}$

(6) $x = \sqrt{4 \cdot 7}$
$x = 2\sqrt{7}$

Step 1: Write the Pythagorean Theorem. Then determine the values of a, b, and c. Remember, c always represents the length of the hypotenuse. In this triangle, the length of the hypotenuse is 8, so c = 8. It does not matter whether a or b is assigned the value of x or the value of 6.
Step 2: Substitute the values of a, b, and c into the Pythagorean Theorem.
Step 3: Following the order of operations, square the 8 (8 x 8 = 64) and the 6 (6 x 6 = 36).
Step 4: Subtract 36 from each side of the equation.
Step 5: Take the square root of each side of the equation.
Step 6: Simplify radical 28. In this case, the variable represents the length of the side of a triangle, so the answer cannot be negative.

Answer: $2\sqrt{7}$.

Solving an Equation by Completing the Square:

Completing the square is a method of solving a quadratic equation in order to express the equation as a single squared term. The method of completing the square is used when an equation cannot be factored. Completing the square involves adding the square of one term to the equation and solving the equation for the value of the variable. The theorem for completing the square states:

To complete the square on $x^2+bx=c$, add $(\frac{1}{2}b)^2$

the result will be $x^2+bx+\left(\frac{1}{2}b\right)^2=c+\left(\frac{1}{2}b^2\right)$ such that

$$\left(x+\frac{1}{2}b\right)^2=c+\left(\frac{1}{2}b\right)^2$$

The following is a detailed example of how to complete the square.

Example 6: Solve the equation by completing the square.

$$x^2+8x-7=0$$

(1) $x^2+8x-7=0$

$$\begin{array}{r} x^2+8x-7=0 \\ +7 \quad +7 \\ \hline x^2+8x=7 \end{array}$$

(2) $b=8$

$$x^2+8x+\left(\frac{1}{2}\cdot 8\right)^2=7+\left(\frac{1}{2}\cdot 8\right)^2$$

$$x^2+8x+4^2=7+4^2$$

(3) $(x+4)^2=7+16$

(3) $\sqrt{(x+4)^2}=\sqrt{23}$

$$(x+4)=\pm\sqrt{23}$$

(5) $x+4=\pm\sqrt{23}$

$$\begin{array}{r} x+4=\pm\sqrt{23} \\ -4 \quad -4 \\ \hline x=-4\pm\sqrt{23} \end{array}$$

(6) $x=-4\pm\sqrt{23}$

Step 1: Add 7 to each side of the equation to put the equation in the form $x^2+bx=c$. Step 2: Since 8 is in the same place as b, b = 8. Add the square of 1/2 of b to each side of the equation. One-half of 8 equals 4, so we are actually adding 4 squared to each side of the equation.
Step 3: We need to fill in the final form in the theorem for completing the square. One-half of b equals 4, so inside the parentheses we have (x + 4). Then we place an exponent of 2 outside the parentheses. Finally, 4 squared equals 16 (4 x 4).

Step 4: First, add 7 and 16 to get 23. Then take the square root of each side of the equation. Taking the square root of a term is the opposite of squaring a term, so we get x + 4 on one side of the equal sign. Remember, every number has a positive and a negative square root, so we write $\pm\sqrt{23}$. Step 5: Subtract 4 from each side of the equation to isolate the x on one side of the equal sign. The new expression cannot be simplified.
Step 6: The solution to the equation is $x = -4 \pm \sqrt{23}$.

Characteristics that Describe the Graph of a Non-Linear Equation:

A quadratic equation is any equation in the form $y = ax^2 + bx + c$. The graph of a quadratic equation is always a parabola. The vertex of a parabola can be found by putting the quadratic equation in vertex form.

Vertex form of a quadratic equation:
$y - k = a(x - h)^2$, where (h, k) is a vertex.

Once a quadratic equation is in vertex form, the vertex is the coordinate point (h, k). If $a > 0$, then the graph opens up and has a minimum. If $a < 0$, then the graph opens down and has a maximum.

The roots of a quadratic equation are the solutions of the quadratic equation when y = 0. The roots are the points where the graph intersects the x-axis; therefore, y = 0. The axis of symmetry is the line that passes through the vertex and splits the graph directly in half such that each side is the mirror image of the other. This line is represented by the equation x = h.

Example 7: What are the characteristics of the graph of $y = -x^2 + 6x + 4$?
Step 1: Rewrite the equation in vertex form. The simplest way to do this is by completing the square.

(A)

$$\begin{array}{r} y = -x^2 + 6x + 4 \\ -4 \qquad\qquad -4 \\ \hline y - 4 = -x^2 + 6x \end{array}$$

(B)

$$y - 4 = -x^2 + 6x$$
$$y - 4 = -1(x^2 - 6x)$$

(C)

$$y - 4 - (\tfrac{1}{2} \cdot -6)^2 = -1(x^2 - 6x + (\tfrac{1}{2} \cdot -6)^2)$$
$$y - 4 - (-3)^2 = -1(x^2 - 6x + (-3)^2)$$

(D)

$$y - 4 - 9 = -1(x - 3)^2$$
$$y - 13 = -1(x - 3)^2$$

Step 1A: Subtract 4 from each side of the equation.
Step 1B: Factor -1 out of the right side of the equation to make the term positive.
Step 1C: Add 1/2 of the b term (-6) squared to the right side of the equation and subtract 1/2 of the b term from the left side of the equation (we subtract on the left side of the equation because we factored -1 out of the equation in Step 1B and we need to multiply any number by -1 before we can add or subtract it from the left side of the equation). b = -6, so we are actually adding -3 squared to the right side of the equation and subtracting -3 squared from the left side of the equation.
Step 1D: Complete the theorem for completing the square.

Step 2: Determine whether the graph opens up or down. Now that the equation is in vertex form, we can determine that a = -1. Since a < 0, the graph opens down.
Step 3: Determine the vertex of the parabola. The vertex is the point (h, k). h = 3 and k = 13, so the vertex is (3, 13).
Step 4: Determine the axis of symmetry. The axis of symmetry is the line x = h. The axis of symmetry is x = 3.
Step 5: Determine the roots of the equation. The roots of the equation are the values of x when y = 0.

(A) $0-13=-1(x-3)^2$

(B) $\dfrac{-13}{-1}=\dfrac{-1(x-3)^2}{-1}$

$13=(x-3)^2$

(C) $\sqrt{13}=\sqrt{(x-3)^2}$

$\pm\sqrt{13}=x-3$

(D) $\pm\sqrt{13}=x-3$

$\underline{+3 \qquad +3}$

$3\pm\sqrt{13}=x$

Step 5A: Substitute 0 in place of y.
Step 5B: Divide each side of the equation by -1.
Step 5C: Take the square root of each side of the equation.
Step 5D: The roots of the equation are $3\pm\sqrt{13}$ (also written $3+\sqrt{13}$ and $3-\sqrt{13}$).
If the 13 had been negative, the equation would have had no real roots.

The characteristics of the graph of $y=-x^2+6x+4$ are:

(1) opens down
(2) vertex is (3, 13)
(3) axis of symmetry is: x = 3
(4) roots are $3\pm\sqrt{13}$ (also written $3+\sqrt{13}$ and $3-\sqrt{13}$).

Spatial Relationships - C

Spatial relationships include understanding geometric transformations as well as recognizing the projection of a three-dimensional object into two dimensions.

Transformations:

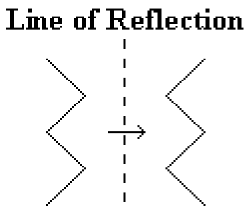
If a figure has changed its size, direction, or position, then the figure has been transformed. The following examples will discuss four different transformations.

Rotation of a figure:

A rotation of a figure changes the direction that a figure is facing. The new figure is found by rotating the original figure about a fixed point for a given number of degrees. The fixed point may be located on or off the original figure.

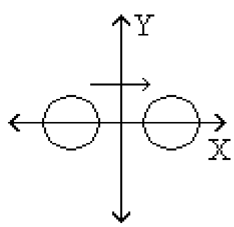
Reflection of a figure:

If two figures are reflections, then they are mirror images of each other. A line of reflection can be drawn between the figures such that if the paper upon which the figures had been drawn was folded on this line, the two figures would coincide. That is, all of the points of one figure would lie upon all of the points of the second figure. If any parts of the figures do not coincide, the figures are not reflections of each other.



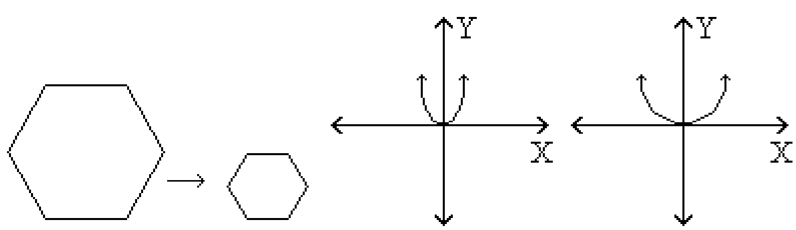
Translation of a figure:

A translation takes a figure and moves it in its entirety along a line from one position to another. Note that after a figure is translated, the figure does not change the direction it is facing, its size, or its shape; only its location has been changed. Watching a train engine move shows a translation in action. As the engine passes you and moves down the track, it changes position, but the size and shape of the train engine do not change. In the following example, the circle has been translated because its position has changed.



Dilation of a figure:

A dilation of a figure can be thought of as the transformation that shrinks or stretches a figure. In the following examples, the hexagon has shrunk in its size whereas the parabola has become "wider."



How to determine the type of transformation:

To determine the type of transformation that has taken place, ask the following questions:

(1) Has the figure changed its size?
If yes, the transformation is probably a dilation.

(2) Has the figure changed its position by travelling along a linear (straight) path?

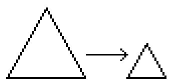
If yes, the transformation is probably a translation.

(3) Do the figures appear to be mirror images of each other?
If yes, the transformation is probably a reflection.

(4) Has the direction the figure was facing changed?
If yes, the transformation is probably a rotation.

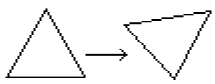
It is possible to combine different types of transformations to make changes to the original figure. The following examples will only involve one transformation.

Example 1: Which transformation was performed on the following figure?



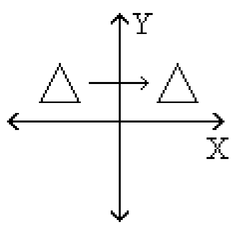
Since the figure has changed its size by getting smaller, the transformation is a dilation.

Example 2: Which transformation was performed on the following figure?



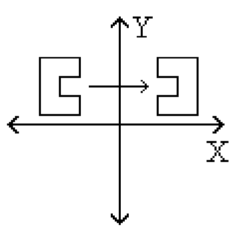
Since the figure has not changed shape, but is facing a different direction, the transformation is a rotation.

Example 3: Which transformation was performed on the following figure?



Since the triangle has shifted to the right along the x-axis, the transformation is a translation.

Example 4: Which transformation was performed on the following figure?



Since the figure appears to have been "flipped" across the y-axis to get its mirror image, the transformation is a reflection.

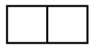
Spatial Relationships (Projection of a three-dimensional object into two dimensions:

The following problems will require the interpretation of two-dimensional pictures drawn of a three-dimensional building. Two pictures of either the top, frontal, right, or left view will be presented and

then questions will be asked about what a viewer sees from one of the other sides.

A top view that looks like figure A below reflects a building that has one change in the height of the building. Figure B shows a building that has two changes in the height of the building.

Top View **Top View**



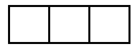
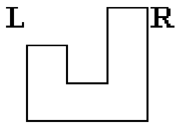


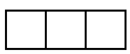
Figure A **Figure B**

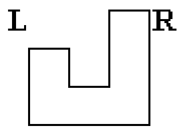
A frontal view such as the one in the diagram below better shows the differing heights of the building. In this example, the building is shown to have three different heights from middle to low to high.



Frontal View

The following are the front and top views of a building to be used for Examples 5 and 6.

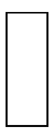




Top View **Frontal View**

Example 5: What is the view from the right side of the building?

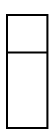
From the right side of the building, the viewer can only see the highest wall. The viewer has no idea if there are any differing heights on the other side of the wall because the view is blocked by the high wall. Thus, the view from the right side of the building is:



Right View

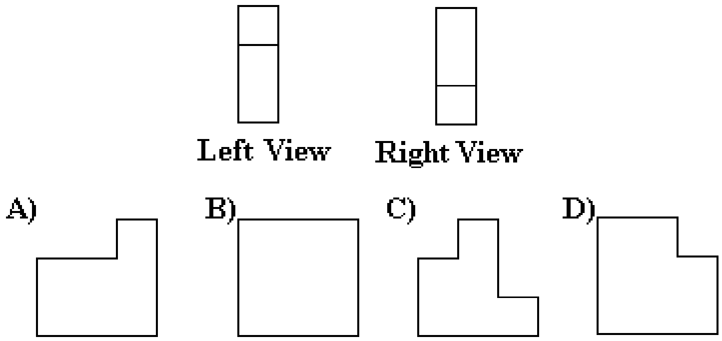
Example 6: What is the view from the left side of the building?

From the left side of the building, the viewer can see two different heights of the building, the middle and highest heights. The "dip" in the building's height cannot be seen by the viewer. Thus the view from the left side of the building is:



Left View

Example 7: The following are the right and left views of a building. Which of the following could be the frontal view?

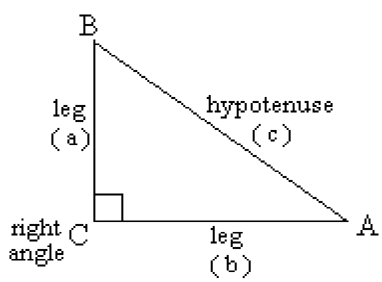


The right view indicates to the viewer that there is at least one change of height with the known change of height at a lower part of the building. The left view indicates to the viewer that there is at least one change of height with the known change of height at a higher level of the building. The viewer cannot tell whether there is more than one change of height because a top view is not presented to confirm the exact number of height changes. Thus the only choice among the four options is Choice C. Note that Choice A does show a height change on the left side, but shows no height change on the right side. Choice B shows no height changes on either the right or left sides. Choice D shows no height change on the left side.

Pythagorean Theorem

The Pythagorean theorem is used to find the lengths of the sides and hypotenuse of a right triangle.

The Pythagorean theorem states that the square of the hypotenuse (the longest side) of a right triangle (a triangle with one 90 degree angle) is equal to the sum of the squares of the legs of that triangle (the two shorter sides).



$$a^2 + b^2 = c^2$$

Looking at the diagram above, we can note that a and b always denote legs of a right triangle and c always denotes the hypotenuse of a right triangle.

Example: The lengths of two legs of a right triangle are 3 and 4. What is the length of the hypotenuse?

(1) $a = 3, b = 4, c = ?$

(2) $3^2 + 4^2 = c^2$

(3) $9 + 16 = c^2$

(4) $25 = c^2$

(5) $\sqrt{25} = \sqrt{c^2}$

$5 = c$

<u>Step 1</u>: Determine the values of a, b, and c. In this case, a and b are known and c is the unknown length of the hypotenuse.

Step 2: Substitute the values of a and b into the Pythagorean theorem.
Step 3: Square 3 (3 x 3 = 9) and square 4 (4 x 4 = 16).
Step 4: Add 9 and 16 to get 25.
Step 5: Take the square root of each side of the equation. The square root of 25 is 5 and the square root of c-squared is c.

Answer: The length of the hypotenuse of this triangle is 5.

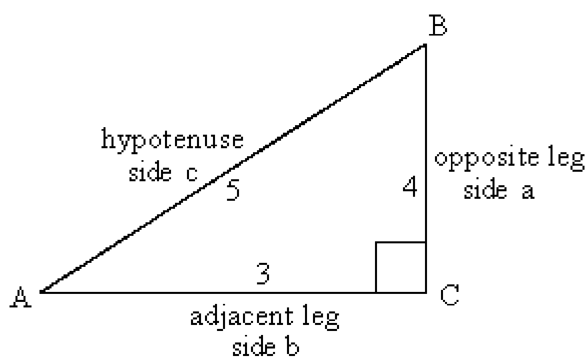
Solving Right Triangles - B

Solving right triangles involves using the trigonometric ratios of sine, cosine, and tangent. Students should be able to use these ratios and trigonometric tables to find the lengths of unknown sides of right triangles and measures of missing angles to the nearest degree.

Trigonometric ratios allow you to find the lengths of the sides of right triangles using one of their acute angles. Acute angles are angles which measure less than 90 degrees. A right triangle has one angle which measures 90 degrees. The triangle side opposite the 90 degree angle is called the hypotenuse. The hypotenuse is the longest side of a right triangle.

Utilize this information to draw a right triangle. Label the 90º angle, ∠C. Label the opposite side, the hypotenuse, side c. Label the acute angle above or below ∠C, ∠B. Label the side across from ∠B, side b. Label the angle to the side of ∠C, ∠A. Label the side opposite ∠A, side a. Now assign measures: side c = 5, side b = 3, side a = 4.

Your triangle should look like the triangle below.



The formulas for trigonometric ratios are:

Sine of an acute angle = measure of the opposite leg/measure of the hypotenuse

Cosine of an acute angle = measure of the adjacent leg/measure of the hypotenuse

Tangent of an acute angle = measure of the opposite leg/measure of the adjacent leg

Therefore, the trigonometric ratios for angle A of the triangle you've drawn are:

sin A = 4/5
cos A = 3/5

tan A = 4/3

Apply this knowledge to the following problem.

Example 1: In right triangle ABC, ∠A measures 28º and Side B measures 21cm. What is the length of Side A?

(1) tan A = Side A/Side B
(2) tan 28º = Side A/21
(3) 0.53 = Side A/21
(4) (0.53 x 21) = Side A
(5) Side A = 11.13

<u>Step 1</u>: Find the trigonometric ratio formula that uses the side and angle information (tangent). Apply the known information to the formula.
<u>Step 2</u>: Look up the tangent of a 28 degree angle on a table of trigonometric ratios. Find the tangent to the nearest tenth degree. (See below for directions to using a table.) The tangent of a 28 degree angle is 0.53. Apply this to the equation.
<u>Step 3</u>: Determine the tangent of 28º and rewrite the equation with that value in place of tan 28º .
<u>Step 4</u>: Cross multiply.
<u>Step 5</u>: Side A of the triangle is 11.13cm.

To read a trigonometric table you need to know either the measure of an acute angle, or the sine, cosine, or tangent of the angle. The following table shows a portion of the Table of Trigonometric Ratios.

Table of Trigonometric Ratios

Angle	Sin	Cos	Tan
12°	0.2079	0.9781	0.2126
13°	0.2250	0.9744	0.2309
14°	0.2419	0.9703	0.2493

The sine (sin) of a 12º angle is 0.2079. The angle degree that has a tangent (tan) of 0.2493 is 14.

There are many real world applications for triangles and using trigonometric ratios and tables to solve right triangles. The following is an example.

Example 2: A flag pole casts a 12 foot shadow at 3:00 p.m., when the angle of elevation of the sun measures 56º . Find the height of the flag pole (side a) to the nearest hundredth.

Draw a flag pole and the sun above. Then, draw a short line from the bottom of the flag pole and make a point (a period). Mark this line as the 12 foot shadow. (Make sure that the sun is on the opposite side of the flag pole as the shadow.) Draw a dotted line from the top of the flag pole to the point to complete the right triangle. Label the side which indicates the flag pole, Side A.

The 90 degree angle is the angle created by the bottom of the flag pole and the line which indicates the flag pole's shadow. The dotted line is the hypotenuse. We know that the angle of elevation of the sun to the flag pole is 56 degrees. This means that the angle created by the point which indicates the 12 foot shadow and the hypotenuse is a 56 degree angle.

(1) tan 56º = Side a/12
(2) 1.48 = Side a/12
(3) (1.48 x 12) = Side a
(4) Side a = 17.76

Step 1: Substitute the known information into the tangent formula.
Step 2: Look up the tangent of 56º in a table of trigonometric ratios and find the measure to the nearest hundredth.
Step 3: Cross-multiply.
Step 4: Determine the length of Side a.

The height of the flag pole to the nearest hundredth is 17.76 feet.